

## Approximation Methods for Solving Differential Equations: Euler's Method

## Lesson 29

"Give me some more Bernoullis," he said. The Marine fighter pilot explained that the expression referred to the lift caused by the curvature of the top of the jet wing. He had seen action in Desert Storm and Vietnam.

We mathematics teachers were touring the Marine Air Base in Beaufort, South Carolina. We climbed a ladder to see the cockpit and panel of instruments, too complicated for my untrained eye. I was in awe of the computations that went into each flight, the technology invested in each jet, the commitment, training, dedication, and courage required of each pilot. I was humbled in his presence.

I was also in awe of the mathematics surrounding me. **Daniel Bernoulli** (1700-1782) would have been proud. He, like his father John Bernoulli, was a great professor of mathematics. His most famous work was with hydrodynamics. Every jet pilot knows the principle of air lift that Bernoulli discovered.

He was a close friend of **Leonhard Euler** (1707-1783). Euler (pronounced oiler) developed the function notation  $y = f(x)$  and contributed to applied physics, astronomy, and shipbuilding. He was a practical guy and solved problems like the one we encounter in this lesson.

## A. The Problem

For each of the following examples, use the technology of today to assist you.

<b>TECH-TIP</b>	[HOME] Screen: [F6] CleanUp: [2] New Problem	page 199
<ul style="list-style-type: none"> <li>Begin each problem:</li> <li>[HOME][CLEAR]: [2nd] [F1], which is [F6] CleanUp: [2] New Problem [ENTER].</li> <li>For more details, see page 199.</li> </ul>		

1.  $\frac{dy}{dx} = -x^2 + 4x - 6$

Initial Point (0, 20)

- a. Use the following TECH-TIP to solve the differential equation. For detailed instructions, see page 199.

<b>TECH-TIP</b>	[HOME] Screen: [F6] Calculus Menu: deSolve(	page 199
<p><b>deSolve</b>(<math>y' = -x^2 + 4x - 6</math> and <math>y(0) = 20, x, y</math>)</p> <ul style="list-style-type: none"> <li>Press [F6] [ENTER] to paste <b>deSolve</b>(</li> <li>Then complete the command as shown above.</li> <li>Press [ENTER] to execute the command.</li> </ul>		

- b. Use the following TECH-TIP to graph the solution in Function Mode. For detailed instructions, see page 200.

<b>TECH-TIP</b>	[HOME] Screen: [F4] Other Menu: [1] Define a Function	page 200
<p>Define <math>y1(x) = -x^3/3 + 2x^2 - 6x + 20</math></p> <ul style="list-style-type: none"> <li>Press [F4] Other: [1] Define:</li> <li>Then complete the command as shown above.</li> <li>Press [ENTER] to execute the command.</li> </ul>		

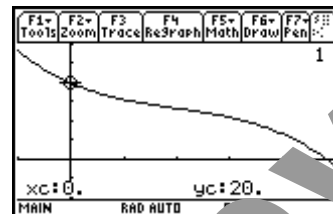
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NOTE

When you define a function on the Home Screen, be very certain that you have:  
 (1) designated which y-function (y1, y2, etc.) and  
 (2) that you have specified that it is a function of x. For example: "y1(x)=" instead of "y1=".

- Press  $\blacktriangleleft$  [F2] for [WINDOW] Editor: X [-1, 5, 1] Y [-10, 30, 5].
- Press  $\blacktriangleleft$  [F3] to access the [GRAPH] Screen.
- Press [F3] to Trace, then [0] [ENTER] to move to the initial point.
- Press [MODE]. Change **Display Digits to Float6:** [ENTER]
- Press  $\blacktriangleleft$  [F1] for the [Y=] Editor. Highlight y1 then press [F4] to turn off the graph.



Repeat the process outlined in #1 to solve the next example. Write your solution in proper Calculus grammar, not calculator syntax.

2.  $\frac{dy}{dx} = (\ln(x))^2 \cdot \sin(x)$  Initial Point (1, 5)

- a. Solve the particular differential equation.

Write your answer in proper mathematical grammar, not in calculator syntax.

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NOTE

In this example, y is the dependent variable and the function is defined as an integral. The independent variable is x. The solution shown on the TI-89 is in terms of t. Ordinarily, this symbol is reserved for constants. In this case, it represents the dummy variable t.

- b. Use the TECH-TIP below to define y2. The graph is in Function Mode. Define  $y2 = \int((\ln(t))^2 * \sin(t), t, 1, x)$  in your calculator.

TECH-TIP

[HOME] screen [F1] Copy: [Y=] Editor:  $\blacktriangleleft$  [ESC] Paste Equation

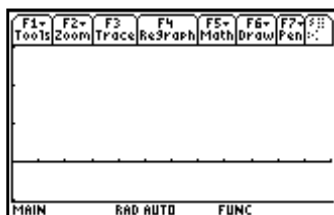
page 200

- Press  $\blacktriangleleft$  to move to the History Area of the Home Screen.
- Press  $\blacktriangleleft$  [F1], which is [COPY] to copy the solution from the previous entry.
- Press  $\blacktriangleleft$  [F1], to move to the [Y=] Editor. [CLEAR] the location for the equation you wish to define.
- Press  $\blacktriangleleft$  [ESC], to [PASTE] the equation into the [Y=] Editor.
- If y1 has an extra "y =", delete that portion from the definition of the function using the  $\blacktriangleleft$  delete key.

Window: X [-5, 12, 1]; Y [-5, 15, 5]

Let xinc = 2 or 3 to make the graphing go faster.

Sketch the resulting graph in the space below.



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**Technology Issue:**

Graphing a function defined as an integral takes more time than usual. Observe the **Busy** signal at the bottom of the **Graph Screen**. Be patient and wait until that signal is gone before you try to move on.

One way to make the graphing go faster is to set **xres** = 3 in your **Window** settings. In doing this, the grapher calculates fewer values than it normally would. You lose a small amount of pixel accuracy; however, it is usually minimal when doing these types of integration problems.

This is a technology issue that you will have to learn to deal with as you gain experience. When is the maximum possible accuracy required; and when does it not make a significant difference in the task at hand?

- c. Find the value of  $y$  when  $x = 2$ . \_\_\_\_\_
3. Explain how your answer to #2a is different from your answer to #1a.

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- a. Could you calculate a table of values and plot points on the graph of #1 with only a scientific calculator? \_\_\_\_\_
- b. Could you calculate a table of values and plot points on the graph of #2 with only a scientific calculator? \_\_\_\_\_

4. Suppose you are given the derivative  $y' = (\ln(x))^2 \cdot \sin(x)$  and the initial point P (2,3), on the graph of  $y = f(x)$ .

- a. Estimate the value of the function when  $x = 2.1$  and explain your methods.  
When you make your argument, remember that you must use proper calculus grammar.

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**NOTE** You must be careful about your mathematical symbols. Be careful to distinguish between different functions. Remember that the value that you get along the tangent line is an **approximation** for the value along the curve,  $y = f(x)$ . You must use the approximation symbol,  $\approx$ , not an equals mark,  $=$ .

- b. Solve and graph the particular differential equation.  $y^3 = f(x) =$  \_\_\_\_\_
- c. Solve for the actual value of the function when  $x = 2.1$ . \_\_\_\_\_

**B. Euler's Method**

In real world applications, differential equations are most often like the one you have just encountered. They do not have an antiderivative that is easily determined. Such was the case during the 17<sup>th</sup> and 18<sup>th</sup> Centuries when scientists and mathematicians were beginning to understand the laws that governed the world in which they lived. They needed a method to solve problems they could not work analytically. It was **Leonhard Euler** who solved the problem.

He started with what was given, the coordinates of a single point. He then calculated the direction given by the derivative at that point. Unfortunately, the instant that you move away from that particular point on the curve, the direction changes. Only when the function is a line, does the direction remain constant.

Euler decided to begin at an initial point and move along a line tangent at that point to a second point. At the new point, the direction should be re-evaluated using the  $x$ - and  $y$ -values in the differential equation. The modified direction leads to another point and the process is repeated.

**Euler's Method**, then, does not actually solve the differential equation. It does, however, plot points which provide an approximation to the graph of the solution to the differential equation. In order to learn his method, begin with a curve that you have already seen.

**1. Make a Table**

$$\frac{dy}{dx} = -x^2 + 4x - 6 \quad \text{Initial Point } (0, 20)$$

Use the following procedure to assist you in making a table of values for the coordinates and the slope at each point.

- Store the initial  $x$ -value as  $x$ . Press  $\boxed{0} \boxed{\text{STO}} \boxed{X} \boxed{\text{ENTER}}$ .
- Store the initial  $y$ -value as  $y$ . Press  $\boxed{2} \boxed{0} \boxed{\text{STO}} \boxed{Y} \boxed{\text{ENTER}}$ .
- Store the derivative,  $-x^2 + 4x - 6$ , as  $m$ . Use  $\boxed{(-)} \boxed{X} \boxed{\wedge} \boxed{2}$  for  $-x^2$ : Use  $\boxed{\alpha} \boxed{5}$  for  $m$ .
- The next point near the solution curve will be a point on the line which is tangent to the curve at the initial point.
- The slope of the tangent line is determined by the derivative evaluated at  $(0, 20)$ .

a. What is the slope of the tangent line at the point  $(0, 20)$ ? \_\_\_\_\_

- The  $x$ -coordinate of the next point will be  $x + \Delta x$ .
- For this example, let  $\Delta x = 0.5$ . Store 0.5 as  $d$ .
- Press  $0.5 \boxed{\text{STO}} \boxed{\alpha} \boxed{5}$ , which is  $d$ . Press  $\boxed{\text{ENTER}}$ .

b. What is the  $x$ -coordinate of the next point?  
Show your calculations.

\_\_\_\_\_

c. Write a formula that you can use on your calculator for calculating the next  $x$ -value.

\_\_\_\_\_

d. What is the  $y$ -coordinate of the next point? Show your calculations.

\_\_\_\_\_

e. Write a formula that you can use for calculating the next  $y$ -coordinate.

\_\_\_\_\_

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- f. The viewing window used for Part A #1 was X [-1, 5]. Since the initial point is at  $x = 0$ , make a table of values for the  $x$ -interval  $[0, 5]$  with step size = 0.5. Also record the slope at each point. Use the formulas that you have developed to assist you.

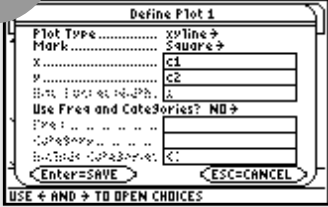
$x$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$y$											
$m$											

2. Plot the Data Points

- a. Follow the TECH-TIP below to create a new Data File in the Data/Matrix Editor. For detailed instructions, see page 201 or pages 255-256.

**TECH-TIP** [APPS]: Data/Matrix Editor: [3] New File: [F2] Plot Setup page 201

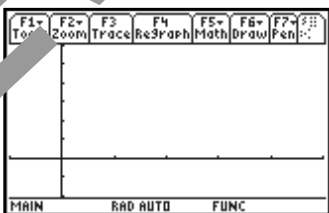
- Select [3] New. Name the Variable file: **eulerdat**.
- Let **c1** =  $x$ -values. Enter them from the table above.
- Let **c2** =  $y$ -values. Enter them from the table above.
- Press [F2] Plot Setup. Highlight Plot1: [F1] Define.
- Define Plot1 as an **xyLine Plot**, Square Mark for (**c1**,
- Press [ENTER] to Save, then [ENTER] to return to the Data Editor.



- b. Sketch the graph and the data plot in the viewing window provided below.
  - Press [F1], for [Y=] Editor.
  - Highlight  $y_1(x)$ . Press [2nd] [F1] [Graph] Styles. Select [2] Dot Style.
  - Press [F2], for the [WINDOW] Editor. X [-1, 5, 1]; Y [-10, 30, 5].

**TECH-TIP** [GRAPH] Screen: [F3] Trace: Along a Data Plot and a Graph page 201

- Press [F3] for the [GRAPH] Screen.
- Press [F1] Trace. Plots are traced first. Points are traced in the order that they were entered.
- Press [F2] to move to the next plot or graph.
- The first point that is traced on a graph is the horizontal mid-point of the graphing screen.
- Use [F1] and [F2] to compare the values of the graph to the corresponding values of the data points.



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c. Euler's Method provides an approximation for the graph of the solution curve of a differential equation. Determine the difference between the  $y$ -values of the data points and the points on the curve.

- Press [APPS]: Highlight Data/Matrix Editor: Press [ENTER].
- Press [1] Current to open the Data File **eulerdat**.

**TECH-TIP** [APPS]: Data/Matrix Editor: [F4] Header: Define a Column page 202

- Within the Data File, move to column 3.
- Press [F4] Header to move to the edit line of the Header.
- Define column 3 as  $y_1(c_1)$ .
- This column contains the  $y$ -values of the function corresponding to the  $x$ -values in column 1.

F1+ Tools	F2 Plot Setup	F3 Ccl	F4 Header	F5 Calc	F6+ Util	F7 Stat
DATA						
		c1	c2	c3		
1	0	20	20			
2	.5	17	17.458			
3	1.	14.875	15.625			
4	1.5	13.375	13.75			
c3=y1(c1)						
MAIN RAD AUTO FUNC						

- Move to column 4 and press [F4] Header.
- Define column 4 as  $c_3 - c_2$ .
- Write the differences in the space below.

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d. Does the data fit the curve exactly? Why or why not?

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Solving Differential Equations: *Text Scripts and Slope Fields*

## Lesson 30

In this lesson, you will create a **Text Script** that will enable you to generate a table of values for Euler's Method for any function. You will be able to edit the function, the initial point, and the step size. The value of the slope at each point, as well as the coordinates of each point will be shown on the **History Area** of the **Home** Screen. By saving it as a **Text File** in your calculator, you can use it again and again.

## A. Create a Text Script

$$\frac{dy}{dx} = -x^2 + 4x - 6 \quad \text{Initial Point } (0, 20)$$

The following procedure is similar to the steps you used in Lesson 29. The keystrokes are given for you on the left and an explanation is given on the right. After you have created the **Text Script**, you will use it to automatically build a data table like the one you created in the last lesson.

Saving this as a **Text File** in your calculator will be useful for you to repeat these steps with minimal effort on other examples in the future.

- In order to make the values for slope, x, and y appear on one line, you need to adjust the number of digits that are displayed on the screen.
- Press **[MODE]**: Change **Display Digits** to **Float4**.

**TECH-TIP****[MODE]**: Changing Display Digits Mode

page 202

**TECH-TIP****[HOME]** Screen: Text Script for Euler's Method

- [HOME]** **[CLEAR]** **[ENTER]** Clear the Entry Line of the Home Screen
- [2nd]** **[F1]**, which is **[F6]**, **[2]** **[ENTER]** New Problem
- [1]** **[STO]** **[alpha]** **[6]**, which is **n**, **[ENTER]** Let **n** represent the number of points
- [(-)]** **[X]** **[^]** **[2]** **[+]** **[4]** **[X]** **[-]** **[6]** **[STO]** **[alpha]** **[5]**, which is **m**, **[ENTER]** Store the derivative  $-x^2 + 4x - 6$  as **m**.
- [0]** **[.]** **[0]** **[STO]** **[X]** **[ENTER]** Store the initial **x**-value, 0.0, as **x**.
- [2]** **[0]** **[.]** **[0]** **[STO]** **[Y]** **[ENTER]** Store the initial **y**-value, 20, as **y**.
- [2nd]** **[{}**, which is **[{]**, **[alpha]** **[,]**, which is **n**, **[alpha]** **[5]**, which is **m**, **[X]** **[,]** **[Y]** **[alpha]** **[,]**, which is **y**, **[ENTER]** Show the initial values:  $\{n, m, x, y\}$
- [0]** **[.]** **[5]** **[STO]** **[alpha]** **[4]**, which is **d**, **[ENTER]** Store 0.5 as **d**.
- [alpha]** **[6]** **[+]** **[1]** **[STO]** **[alpha]** **[1]**, which is **n+1**, **[ENTER]** Calculate values for next point, **n+1**.
- [Y]** **[+]** **[alpha]** **[1]** **[STO]** **[Y]** **[ENTER]** Calculate new **y**-value,  $y_{old} + m_{old} \cdot d \rightarrow y_{new}$ .
- [X]** **[+]** **[alpha]** **[1]** **[STO]** **[X]** **[ENTER]** Calculate new **x**-value,  $x_{old} + d \rightarrow x_{new}$ .
- Press **[↵]** 10 times, then **[ENTER]** **[ENTER]** Show new values:  $\{n, m, x, y\}$
- Copy the next 4 steps into one:
  - Press **[↵]** 8 times, **[ENTER]** Cut and paste **n+1** for next point
  - then **[2nd]** **[4]** Colon : connects two commands
  - Press **[↵]** 6 times, **[ENTER]** Cut and paste calculation of **y**-value
  - then **[2nd]** **[4]** Colon : connects two commands
  - Press **[↵]** 4 times, **[ENTER]** Cut and paste calculation of **x**-value
  - then **[2nd]** **[4]** Connect another command
  - Press **[↵]** 2 times, **[ENTER]** Cut and paste display of new values
  - [ENTER]** Executes the commands.
- Only the last command results are shown.
- Press **[ENTER]** until you reach **x = 5.0**. Calculate new values.
- Press **[F1]** Tools **[2]** Save Copy As... Save as a Text File.
- [↵]** Enter **eulertbl** Name the Text File Variable.
- Press **[ENTER]** Save the Variable name.
- Press **[ENTER]** again. Save the Text File.

Solving Differential Equations: Text Scripts and Slope Fields

Lesson 30

B: Use Slope Fields


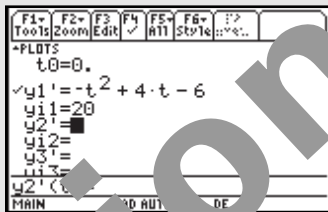
$$\frac{dy}{dx} = -x^2 + 4x - 6 \quad \text{Initial Point } (0, 20)$$

Solve for  $f(x) = 20 + \int_{t=0}^x f'(t) dt$

**TECH-TIP**      **[MODE]: Differential Equations: [Y=] Slope Field with Initial Conditions**      page 201

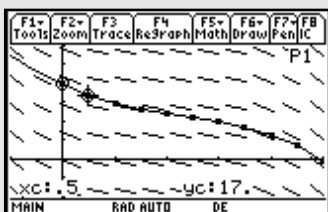
Use the full capability of the TI-89 by plotting a slope field with the initial conditions (0, 20).

- Change **Graph Mode** to **Differential Equations**.
- [F1]**, **[F1]** **[8]** **[ENTER]**      Clear all functions in [Y=]
- [F1]** **[9]**      Format as shown on the screen below.
- [ENTER]**      To Save
- In **Differential Equation Mode**, the expression for y-prime is entered in terms of  $t$ . This agrees with standard mathematical notation for functions defined as an integral. The variable  $t$  is a dummy variable. **The function that you enter in terms of the variable  $t$  is not graphed. It is the derivative of the function that we seek.**
- Enter the equation  $f'(t)$  as shown on the screen.

- [F2]**, [WINDOW]:  $t$  [0, 5, 0.5, 0];  $x$  [-1, 5, 0.5, -10, 5, 5] ncurves=0, Estep=1; fieldres=12.
- [F3]**, which is [GRAPH].
- [F3]** Trace and compare values to those of your table of values in Lesson 29.

In the **Window** variables, **tstep** = 0.5 is equivalent to  $\Delta x$ . The value assigned to  $t_0$  is the initial  $x$ -value. The value assigned to  $yi$  is the initial  $y$ -value and is optional. In the graph, the initial point is marked with a circle. If no initial  $y$ -value is given, only the **slope field** is shown. Trace on the solution curve and compare the values to those of the **xyLine Plot** from Lesson 29.



- How do the values in your table compare to the values plotted by the calculator in **Differential Equation Mode**?

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- Make a conjecture that could explain your answer to #1.

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**Solving Differential Equations: Text Scripts and Slope Fields**

**Lesson 30**

2. Graph and Slope Field

$\frac{dy}{dx} = (\ln(x))^2 \cdot \sin(x)$  Initial Point (5, 4)

Use the viewing window X[0, 12, 1]; Y [-5, 15, 5].

a. Select other Window variables so that plotted values will agree with the values in your table above.

- tstep = \_\_\_\_\_ ncurves = 0
- t<sub>0</sub> = \_\_\_\_\_ tplot = \_\_\_\_\_
- y<sub>i</sub> = \_\_\_\_\_ fldres = 14 is standard

b. Graph the solution to the differential equation in the screen provided.

- Circle the initial point.
- According to your graph, when  $x = 5.3$ ,  $y =$  \_\_\_\_\_.

c. The screen above shows the slope field for the differential equation. Why was it helpful to you as you sketched the graph in #2b?

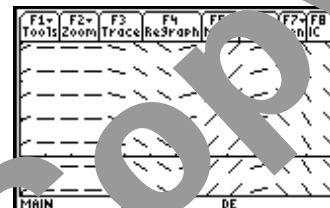
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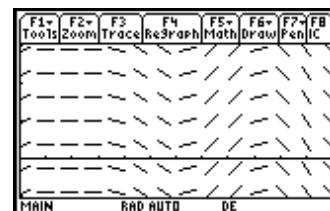
d. Follow the directions below to graph the solution with initial conditions from the graph screen.

- Press  $\blacklozenge$  [F1], to access the [Y=] Editor. Highlight the entry for y1 and press [CLEAR].



<b>TECH-TIP</b>	<b>[MODE]: Differential Equations: [GRAPH] [F8] Interactive Initial Conditions</b> <span style="float: right;">page 205</span>
<p>Interactively enter the initial conditions (5) and (1, 5):</p> <ul style="list-style-type: none"> <li>• Press <math>\blacklozenge</math> [F3], to access the [GRAPH] screen.</li> <li>• Press <math>2^{nd}</math> [F3], which is [F8], Initial Conditions.</li> <li>• Press [6] [ENTER] for the x-coordinate.</li> <li>• Press [5] [ENTER] for the y-coordinate.</li> </ul>	

Sketch the resulting graph in the screen provided. Circle the initial point.

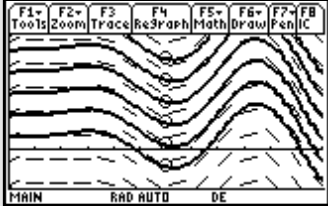


**Solving Differential Equations: Text Scripts and Slope Fields**

3.  $\frac{dy}{dx} = (\ln(x))^2 \cdot \sin(x)$  Initial Point (a, b)

**TECH-TIP** [MODE]: Differential Equations: [WINDOW] ncurves: Automatic Solutions page 205

If you do not specify an initial point, the calculator can automatically draw up to 10 solution curves for you. This is determined in the Window variable **ncurves**. The default is **ncurves = 0**.



- a. Reproduce this on your calculator as follows:
- In the [Y=] Editor, be sure that you have cleared  $y_1$ .
  - In the [WINDOW], let **ncurves = 6**.

The curves are evenly distributed along the y-axis and  $t0$  is temporarily set to the middle of the screen. Since these curves are drawn, you will not be able to trace along these curves.

b. Solve:  $\frac{dy}{dx} = (\ln(x))^2 \cdot \sin(x)$  Initial Conditions P(a, b)  $y =$  \_\_\_\_\_

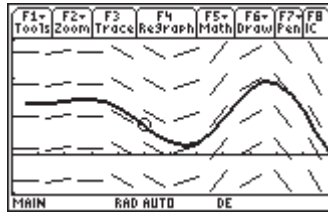
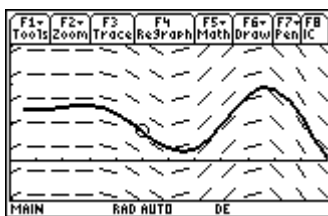
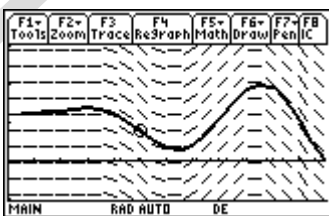
Write the solution to the differential equation as an integral expression using the initial conditions.

- c. How does the screen in #3a illustrate the expression in #3b? \_\_\_\_\_

**Slope fields:** A method for visualizing solutions to differential equations.  
**Plotting a field of slopes:** Calculators use a problem solving method that you should use.  
**The Method:** Start with what you know; then see where it leads you.

**Given:**  $\frac{dy}{dx} = (\ln(x))^2 \cdot \sin(x)$  Initial Point (a, b)

4. Write two things that you know when solving this differential equation.
- a. \_\_\_\_\_
- b. \_\_\_\_\_
- c. What does that tell us graphically? \_\_\_\_\_
- d. We can construct a short segment at the point (a, b) that represents the line that is \_\_\_\_\_ to a solution curve at that point.
- e. In your calculator, what determines how many segments are plotted in the slope field?



**Solving Differential Equations: Text Scripts and Slope Fields**

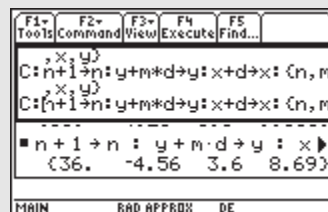
5. Defining a slope field

- a. Complete this statement:  
A slope field consists of \_\_\_\_\_.
- b. How does the screen in #3a illustrate the Fundamental Theorem of Calculus?  
\_\_\_\_\_  
\_\_\_\_\_
- c. Explain what a slope field is. How does it illustrate the points outlined in #3?  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Evaluation Copy

**Journal Entries**

1. Euler's Method begins at the \_\_\_\_\_.
2. The second point is determined by moving along \_\_\_\_\_.
  - a. the curve
  - b. a tangent line
  - c. a horizontal line
3. What property of functions must exist in order to use Euler's Method?
  - a. Mean-Value Theorem of Derivatives
  - b. Fundamental Theorem of Calculus
  - c. Local Linearity
4. Explain your answer to #3.
5. True or False. If false, state the reason why it is not true.  
Euler's Method gives a general formula for  $y$  in terms of  $x$  that is the particular solution of a differential equation.
6. The output of the Text Script for Euler's Method below is  $\{n, m, x, y\}$ , where  $n$  represents the number of points calculated,  $m$  represents the slope at the point  $(x, y)$ , and  $(x, y)$  are the coordinates of the next point if  $\Delta x = 0.1$ . Show your calculations.
7. Name the two methods of approximating solutions to differential equations that you have seen in this chapter. What is their relationship?
8. Suppose that you are given an initial point  $A(3, 5)$ . You are asked to use Euler's Method to approximate a  $y$ -value at point  $P(a, b)$ . Name two things that affect the accuracy of an approximation for a  $y$ -value using this method.
9. Solve:  $\frac{dy}{dx} = f'(x)$  Initial Conditions  $P(a, b)$



Write the solution to the differential equation as an integral expression using the initial conditions.

**Chapter 6: TECH-TIPS**

**Lesson 29 TECH-TIPS**

**TECH-TIP** [HOME] Screen: [F6] CleanUp: [2] New Problem

Begin each new section with a **NewProb** command.  
Press [HOME] [CLEAR] to clear the **Entry Line** of the **Home Screen**.

[2nd] [F1], which is [F6]

[2] **NewProb**

[ENTER] to execute.

This command

- (1) clears all values stored in single-letter variables.
- (2) clears the **History** area of the **Home Screen**.
- (3) turns off all **Data Plots** and graphs in the [Y=] editor.

This does not clear the equations or plot definitions; however it does turn them off so that they will not be graphed.

If you only want to clear the **History** area of the **Home Screen**, press [F1]Tools, [8]Clear Home.

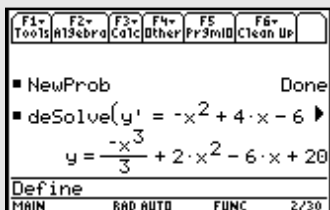
**TECH-TIP** [HOME] Screen: [F3] Calculus Mode: deSolve

Enter **deSolve**( $y' = -x^2 + 4x - 6$  and  $y(0) = 20$ ,  $x$ ,  $y$ )

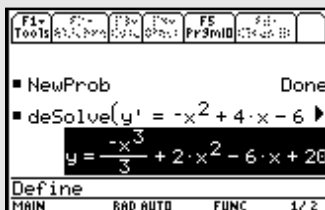
- Press [F3] [ENTER] to paste the command **deSolve**( on the **Entry Line**.
- Enter the differential equation.
- Use  $y$ -prime, [Y] [2nd] [F1], instead of  $dy/dx$  for the derivative, [F1].
- Complete the differential equation.
- Press [ALOG] [ENTER] to paste “and” in the **Entry Line**.
- Next, enter the initial conditions in function notation,  $y(0)=20$ , then [F1].
- The last parameters needed are the independent variable [X], then the dependent variable [Y], separated by comma followed by closing parenthesis [F1].
- Press [ENTER] to see the solution to the differential equation.

**TECH-TIP** [HOME] Screen: [F4] Other Menu: [1] Define a Function

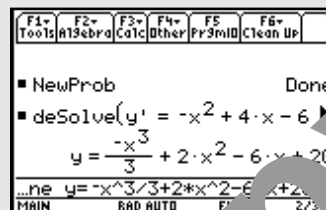
- [HOME][CLEAR], to clear the Entry Line.



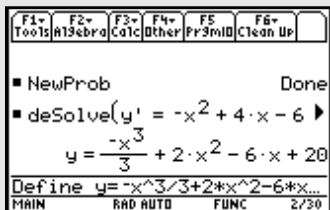
[F4] Other, [1] Define



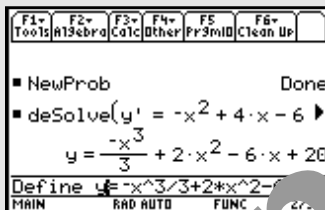
↶



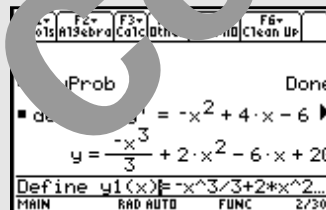
[ENTER] Pastes the equation into the Entry Line



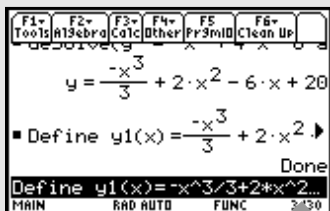
[2nd] [↶] Move the cursor to the beginning of the line.



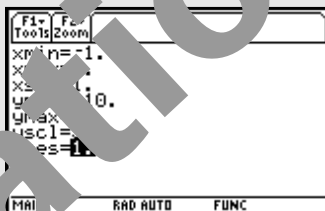
Press [↷] until the cursor is positioned after y.



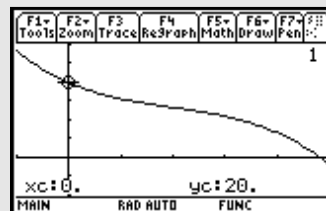
[1] [ ] [X] [ ]



[ENTER]



[2], which is [WINDOW].  
X: [-1, 5, 1] Y: [-10, 30, 5]



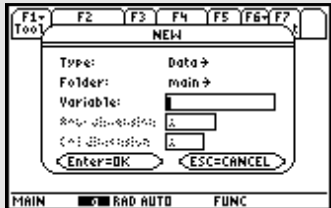
[F3], which is [GRAPH].  
[F3] Trace: [0] [ENTER].

**TECH-TIP** [HOME] Screen: [↵][↑] Copy: [Y=] Editor: [↵][ESC] Paste Equation


- Press [↶] to move to the History Area of the Home Screen.
- Press [↵][↑], which is [COPY] to copy the solution from the previous entry.
- Press [Y=], to move to the [Y=] Editor. [CLEAR] the location for the equation you wish to define.
- Press [↵][ESC], to [PASTE] the equation into the [Y=] Editor.
- If you pick up an extra "y =", delete that portion from the definition of the function using the [←] delete key.

**Chapter 6: TECH-TIPS**

**TECH-TIP** [APPS]: Data/Matrix Editor: [3] New File: [F2] Plot Setup



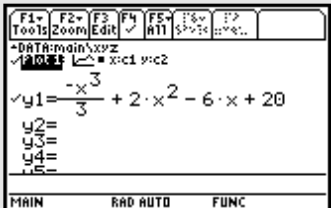
DATA	c1	c2	c3
1	0	20	
2	.5	17	
3	1.	14.88	
4	1.5	13.38	

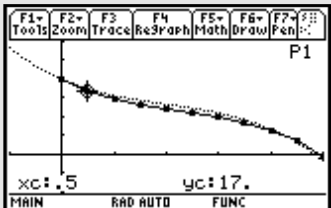


- Press [APPS][6], Data/Matrix Editor, [3]New.
- Name the Variable: **eulerdat**.
- Then press [ENTER] twice.
- In column 1, enter the x-values from the table.
- If you are in **Auto Mode**, be sure to include a decimal in your numerical entry.
- In column 2, enter the y-values from the table.
- Press [F2] Plot Setup, highlight Plot.
- Press [F1] Define as **xyLine** Plot, using the **Square Mark**, **Auto** for x, and **Auto** for y.
- Press [ENTER] to move and [ENTER] to return to the data file.

**TECH-TIP** [GRAPH] Screen: [F3] Trace: Along a Data Plot and a Graph

- Press [F1], which is [Y=] Editor. Highlight the equation for y1.
- Press [2nd][F1], which is [F6]Style. Select [2]Dot.
- Press [F2], which is [WINDOW]: X [-1, 5, 1]: [-1, 30, 5], xres = 3.
- Press [F3], which is [GRAPH].
- Press [F3] Trace. Press [↑] to move along the xyLine Plot. When you trace on a plot, the data points appear in the corner that they are entered in the Data/Matrix. When you get to the last point, press [↓] to re-trace the points.
- Press [←] to move to the graph. Compare the values of the graph to the data recorded in the table. When you initially trace on a graph, the cursor moves to the middle of the screen. Use [←] and [↑] to move along the graph.
- In order to move to a specific point on the graph of a function, press [F3] trace, enter the x-coordinate of the desired point, then press [ENTER]. The cursor moves to that point. Notice that the x- and y-coordinates are shown at the bottom of the graphing screen. **This method does not work on a Data Plot.**





Chapter 6: TECH-TIPS

TECH-TIPS

TECH-TIP

[APPS]: Data/Matrix Editor: [F4] Header: Define a Data Column

- Move to column 1, then press [F4] Header to move to the Edit Line of the header.
- Press [2nd] [5], for [MATH], [3] List, [1] seq(
- You want the column to contain a sequence of numbers of the form  $x$ , with respect to the variable  $x$ , beginning with zero, ending with 5.0, in increments of 0.5.
- It is not necessary to use the variable  $x$ . Any alpha-variable will work. The variable  $x$  is chosen here for convenience.
- Entering the definition of column 1 in this manner makes it easy to change to smaller increments or to change either the beginning or ending values.
- When values in column 1 are changed, the values of any columns that are calculated in terms of column 1 are automatically updated.
- Notice that individual cells in column 1 are therefore locked (see 2<sup>nd</sup> screen below) and can be changed only if the definition of the entire column is changed. (See third screen below.)

F1- Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA						
		c1	c2	c3		
1		0				
2		.5				
3		1.				
4		1.5				
c1=seq(x,x,0,5,.5)						
MAIN RAD AUTO DE						

F1- Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA						
		c1	c2	c3		
1		0				
2		.5				
3		1.				
4		1.5				
a1:c1=0						
MAIN RAD AUTO DE						

F1- Tools	F2 Plot Setup	F3 Cell	F4 Header	F5 Calc	F6 Util	F7 Stat
DATA						
		c1	c2	c3		
1		0	20			
2		.5	9.42			
3		1.	3.877			
4		1.5	8.371			
c2=4/c1						
MAIN RAD AUTO FUNC						

Lesson 30 TECH-TIPS

TECH-TIP

[MODE]: Changing Display Digits Mode

- Press [MODE], then  $\leftarrow$   $\rightarrow$  to move to the Display Digits line.
- Press  $\downarrow$  to see the Menu, then press  $\leftarrow$  for Matrix Mode or  $\rightarrow$  for Float Mode.
- Highlight your selection and press [ENTER].
- Press [ENTER] again to Save changes in the Mode Screen.

F1 Page 1	F2 Page 2	F3 Page 3
MODE		
Graph.....	H:FLOAT	3
Current Folder.....	F:FLOAT	4
Display Digits.....	J:FLOAT	5
Ans1.....	K:FLOAT	6
Exponential Format.....	L:FLOAT	7
Complex Format.....	M:FLOAT	8
Vector Format.....	N:FLOAT	9
Pretty Print.....	O:FLOAT	10
Enter=SAVE		
TYPE OR USE $\leftarrow$ +1 + (ENTER) OR (ESC)		

[HOME] Screen: Create a Text File

Create the commands on the Home Screen, then save them as a Text File. Saving a **Text File** in your calculator will be useful for you to repeat the same steps with minimal effort for other examples in the future.

Usually, it is a good idea to begin with a **New Problem** command or **Clear a-z**. If you want functions graphed, turn them on and select appropriate Window variables.

- [HOME] [CLEAR] Clear the Entry Line of the Home Screen
- [2nd][F1], which is [F6], [2][ENTER] New Problem (or select 1: Clear a-z)

## Chapter 6: TECH-TIPS

**TECH-TIP** [HOME] Screen: [F1] Tools: [2] Save Copy As...A Text File

Press  $\leftarrow \rightarrow$  several times to review the calculations that you used in a previous example. The calculations are preserved in the **History Area** of the **Home** Screen. These repetitive keystrokes can be saved in a **Text File** and used with other examples.

- Press [F1] Tools [2] Save Copy As . . .
  - $\leftarrow$  Enter **eulertbl**
  - Press [ENTER]
  - Press [ENTER] again.
- Save as a Text File.  
Name the Text File Variable.  
Save the Variable name.  
Save the Text File.

**TECH-TIP** [APPS]: Text Editor: Using a Text File**Opening the File**

- [APPS] Text Editor: [ENTER]: [2] Open
- Press  $\leftarrow$  to highlight the Variable field.
- Press  $\rightarrow$  to open the list of Text Files.
- Press  $\leftarrow$  to highlight the Text File Variable **eulertbl**.
- Press [ENTER] [ENTER] to open the file.

**Inside the Text File**

- $\uparrow \leftarrow$  takes you to the top of the file.
- For a new example, edit the expression for  $m$ , and the initial values for  $x$  and  $y$ .
- Let  $\Delta t = d = 0.1$

**Command Keys**

- [F3] [1] Script view – Horizontal split screen

The Text Script appears in the top half of the view screen and another application in the bottom half of the screen. For example, the second application could be a graph, a table, or the Home screen. The result of each command is viewed on the second application as it is executed in the Text Script.

- [F4] Executes each command line.
- [2nd] [APPS], which is [Esc] Alternates between applications.
- [2nd] [Esc], which is [QUIT] Exit the Text File.

Chapter 6: TECH-TIPS

TECH-TIPS

**TECH-TIP** [MODE]: Differential Equations: [Y=] Slope Field with Initial Conditions

Given:  $\frac{dy}{dx} = -x^2 + 4x - 6$  Initial Point (0, 20)

Solve:  $f(x) = 20 + \int_{t=0}^x f'(t) dt$

Use the full capability of the TI-89 by plotting a slope field with the initial conditions (0, 20). This can be done by changing the Graph Mode to Differential Equations.



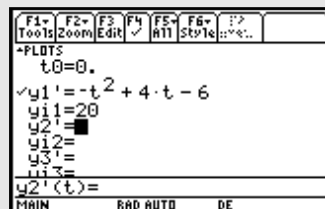
- [MODE] [6] [ENTER]
- [F1] [F1] [8] [ENTER]
- [F1] [9]
- [ENTER]

Change Mode to Differential Equations. Clear all functions in [Y=] Editor. Graph Format as shown below. To save changes.

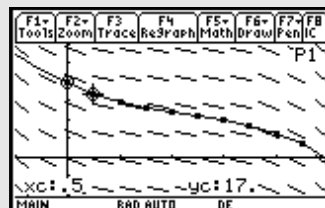
In **Differential Equation Mode**, the expression for y-prime is entered in terms of  $t$ . This agrees with standard mathematical notation for functions defined as an integral. The variable  $t$  is a dummy variable. **The function that you enter in terms of the variable  $t$  is not graphed. It is the derivative of the function that we seek.** Enter the equation as shown on the screen.



- [F2], [WINDOW]:  $t$  [0, 5, 0.5, 1];  $x$  [-1, 5, 1];  $y$  [-10, 30, 5] ncurv=1, Estep=1, fldres=12
- [F3], which is [GRAPH].



In the Window variables,  $tstep = 0.5$  is recommended to use. Usually in Euler's Method, a  $tstep$  of 0.1 is used. The value assigned to  $t_0$  is the initial  $x$ -value. The value assigned to  $y_i$  is the initial  $y$ -value and is optional. On the graph, the initial point is marked with a circle. If an initial  $y$ -value is given, only the slope field is shown.



## Chapter 6: TECH-TIPS

## TECH-TIPS

## TECH-TIP

[MODE]: Differential Equations: [GRAPH]: [F8] Interactive Initial Conditions

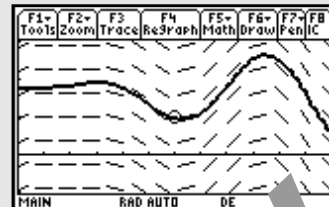
Given:  $\frac{dy}{dx} = (\ln(x))^2 \cdot \sin(x)$  Initial Point (6, 5)

Solve:  $y = 5 + \int_{t=6}^{t=x} [y'(t)] dt = 5 + \int_{t=6}^{t=x} (\ln(t))^2 \cdot \sin(t) dt$

Use the viewing window X[0, 12, 1] Y [-5, 15, 5].

To interactively enter the initial conditions (6, 5):

- Press **2nd** **F3**, which is **F8**, Initial Conditions.
- Press **6** **ENTER** for the  $x$ -coordinate.
- Press **5** **ENTER** for the  $y$ -coordinate.



## TECH-TIP

[MODE]: Differential Equations: [WINDOW] ncurves Automatic Solutions

Given:  $\frac{dy}{dx} = (\ln(x))^2 \cdot \sin(x)$  Initial Point (a, b)

Solve:  $y = b + \int_{t=a}^x [y'(t)] dt$

If you do not specify an initial point, the calculator can automatically draw up to 10 solution curves for you. This is determined by the Window variable **ncurves**. The default is **ncurves** = 0.

- In the **[Y=]** Editor, be sure that you have cleared the screen.
- In the **[WINDOW]**, let **ncurves** = 6.
- Use the viewing window X[0, 12, 1] Y [-5, 15, 5].

The curves are evenly distributed along the  $y$ -axis and  $t0$  is temporarily set to the middle of the screen. After these curves are drawn, you will not be able to trace along these curves.

